Class XII Session 2025-26
Subject - Applied Mathematics
Sample Question Paper - 2

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If A is a square matrix of order 3, such that A(adjA) = 10I, then |adj A| is equal to **[1]**

a) 1000 b) 10

c) 100 d) 1

2. For the purpose of t-test of significance, a random sample of size (n) 34 is drawn from a normal population, then the degree of freedom (v) is **[1]**

a) $\frac{1}{34}$ b) 33

c) 35 d) 34

3. Mr. X borrowed ₹ 500000 from a bank to purchase a house and decided to repay the loan by equal monthly payments in 10 years. If bank charges interest at 7.5% p.a. compounded monthly, then EMI is: (Given $(1.00625)^{120} = 2.1121$) **[1]**

a) ₹ 8520 b) ₹ 6380

c) ₹ 5935 d) ₹ 7340

4. Solution set of inequations $x - 2y \geq 0$, $2x - y \leq -2$, $x \geq 0$, $y \geq 0$ is: **[1]**

a) Closed halfplane
b) First quadrant
c) Empty
d) Infinite

5. What is x if $\begin{bmatrix} 1 & 4 \\ 2 & x \end{bmatrix}$ is a singular matrix? **[1]**

a) 5
b) 6
c) 8
d) 7

6. In a binomial distribution, the probability of getting success is $\frac{1}{4}$ and standard deviation is 3. Then, its mean is **[1]**

a) 8
b) 6
c) 12
d) 10

7. If X follows a binomial distribution with parameters n = 100 and p = $\frac{1}{3}$, then P (X = r) is maximum when r = **[1]**

a) 33
b) 32
c) 34
d) 31

8. The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when y(0) = 2 is **[1]**

a) three
b) one
c) two
d) infinite

9. A man rows d km upstream and back again in t hours. If he can row in still water at u km/hr and the rate of stream is v km/hr, then t = **[1]**

a) $\frac{u^2-v^2}{d}$
b) $\frac{uv}{d}$
c) $\frac{2ud}{u^2+v^2}$
d) $\frac{2ud}{u^2-v^2}$

10. In a 500 m race, the ratio of speeds of two contestants A and B is 3 : 4. If A gets a start of 140 m, then he wins by: **[1]**

a) 10 m
b) 20 m
c) 60 m
d) 40 m

11. The least positive integer x satisfying 28 $\equiv$ x (mod 6) is **[1]**

a) 4
b) 2
c) 1
d) 3

12. The graph of the inequation 2x + 3y > 6 is the: **[1]**

a) half-plane that neither contains the origin nor the points on the line 2x + 3y = 6
b) whole XOY-plane excluding the points on the line 2x + 3y = 6
c) half-plane that contains the origin
d) entire XOY-plane

13. A pipe can fill a tank in 3 hours. Because of a leak in the tank the pipe takes 3 hours 30 minutes to fill the tank. How much time will the leak take to empty the full tank? **[1]**

a) $\frac{1}{2}$ hours
b) 21 hours
c) 7 hours
d) $6\frac{1}{2}$ hours

14. In an L.P.P. if the objective function Z = ax + by has same maximum value on two corner points of the feasible region, then the number of points at which the maximum value of Z occurs is **[1]**

a) infinite
b) 2
c) finite
d) 0

15. $\int \frac{dx}{x(x^7+1)}$ is equal to **[1]**

a) $\log\left|\frac{x^7+1}{x^7}\right| + C$
b) $\log\left|\frac{x^7}{x^7+1}\right| + C$
c) $\frac{1}{7}\log\left|\frac{x^7}{x^7+1}\right| + \mathrm{C}$
d) $\frac{1}{7}\log\left|\frac{x^7+1}{x^7}\right| + \mathrm{C}$

16. Since α = probability of Type-I error, then 1 - α **[1]**

a) Probability of rejecting H_0 when H_a is true.
b) Probability of not rejecting H_0 when H_0 is true.
c) Probability of not rejecting H_0 when H_a is true.
d) Probability of rejecting H_0 when H_0 is true.

17. $\int(x - 1)e^{-x}$ dx is equal to **[1]**

a) $-xe^{-x} + C$
b) $(x + 1)\, e^{-x} + C$
c) $xe^{-x} + C$
d) $(x - 2)\, e^{-x} + C$

18. An orderly set of data arranged in accordance with their time of occurrence is called: **[1]**

a) Time Series
b) Arithmetic Series
c) Harmonic Series
d) Geometric Series

19. Let A be a non-singular matrix of order n. **[1]**

Assertion (A): adj(adj A) = $|A|^{n-2}$ A

Reason (R): |adj A| = $|A|^{n-1}$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The slope of the normal to the curve $y = 2x^2 - 5x$ at x = -1 is -1. **[1]**

Reason (R): The slope of the normal to the curve y = f(x) at point (α, β) is given by $(x - \alpha) + \left(\frac{dy}{dx}\right)_{(\alpha,\beta)} \cdot (y - \beta) = 0$.

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
c) Assertion (A) is true but Reason (R) is false.
d) Assertion (A) is false but Reason (R) is true.

Section B

21. Construct 3-yearly moving averages from the following data: **[2]**

Year:	2010	2011	2012	2013	2014	2015	2016
Imported cotton consumption in India (in '000 bales):	129	131	106	91	95	84	93

22. A man wants to deposit a lump sum amount so that an annual scholarship of ₹ 3000 is paid. Rate of interest is **[2]**

5% per annum. Calculate the lump sum amount required, if the scholarship is to start at the end of this year and continue forever.

OR

A simple interest of ₹ 1000 is paid on a certain sum of money at 10% p.a for 4 years. Find the sum.

23. Evaluate: $\int_1^2 \frac{\log x}{x^2}$ dx **[2]**

24. If A' = $\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find (A + 2B)'. **[2]**

OR

If the matrix A = $\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew-symmetric, find the values of a and b.

25. How many kg of sugar costing ₹ 45 per kg must be mixed with 30 kg sugar costing ₹ 35 per kg so that there may be a gain of 12% by selling the mixture at ₹ 47.04 per kg? **[2]**

Section C

26. It is given that radium decomposes at a rate proportional to the amount present. If p % of the original amount of radium disappears in 1 year. What percentage of it will remain after 2l years? **[3]**

OR

Solve: $(x^2 + 1)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to the initial condition y(0) = 0.

27. The cost of a washing machine depreciates by ₹720 during the second year and by ₹648 during the third year. Calculate: **[3]**

i. the rate of depreciation per annum.
ii. the original cost of the machine.
iii. the value of the machine at the end of third year.

28. The marginal cost of production of x units of a commodity is 30 + 2x. It is known that fixed costs are ₹ 120. Find **[3]**

i. the total cost of producing 100 units
ii. the cost of increasing output from 100 to 200 units.

29. The probability distribution of a discrete random variable X is given as under: **[3]**

X	1	2	4	2A	3A	5A
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Calculate:

i. The value of A, if E(X) = 2.94
ii. Variance of X

OR

In a group of 30 scientists working on an experiment, 20 never commit error in their work and are reporting results elaborately. Two scientists are selected at random from the group. Find the probability distribution of the number of selected scientists who never commit error in the work and reporting. Also, find the mean of the distribution. What values are described in the question?

30. Fit a straight line trend by the method of least squares to the data given below: **[3]**

Years	2012	2013	2014	2015	2016	2017	2018
Sales (in tones)	9	11	13	12	14	15	17

31. A group of 5 patients treated with medicine A weigh 10, 8, 12, 6, 4 kg. A second group of 7 patients treated with medicine B weigh 14, 12, 8, 10, 6, 2, 11 kg. Comment on the rejection of hypothesis with 5% level of significance. **[3]**

[Given: $t_{(10,0.05)} = 1.812$]

Section D

32. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If the profit on a racket and on a bat is ₹20 and ₹10 respectively, find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make it as an LPP and solve it graphically. **[5]**

OR

A fruit grower can use two types of fertilizer in his garden, brand P and Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

kg per bag		
	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

33. A company manufactures cassettes and its cost and revenue functions for a week are C = 300 + $\frac{3}{2}x$ and R = 2x respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold for the company to realize a profit? **[5]**

34. A radar unit is installed to measure the speeds of cars on a highway. The speeds are normally distributed with mean 80 km/h and standard deviation 10 km/h. If a car is chosen at random, find the probability that car is running **[5]**

   i. at less than 60 km/h
   ii. at more than 100 km/h
   iii. between 90 km/h and 110 km/h.

OR

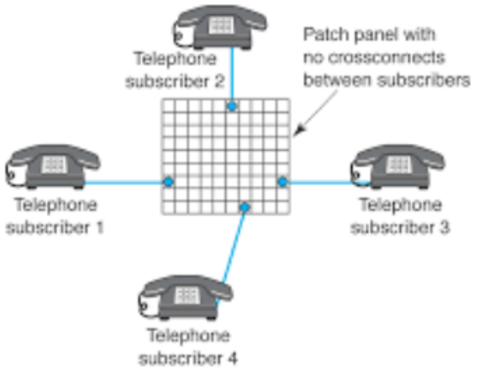
An urn contains 5 red and 2 black balls. Two balls are randomly drawn without replacement. Find the probability distribution of the black balls drawn. Also, find the mean and variance of the black balls drawn.

35. Mrs. Jain is considering to buy a ₹ 1,000 par value bond bearing a coupon rate of 11% that matures after 5 years. She wants a minimum rate of return of 15%. The bond is currently sold at ₹ 870. Should she buy the bond? Justify your answer. [Given: $(1.15)^{-5} = 0.4971$] **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A telephone company in a town has 500 subscribers on its list and collects fixed charges of 300 per subscriber per year. The company proposes to increases the annual subscription and it is believed that for every increase of ₹ 1 one subscriber will discontinue the service.



(a) If x be the annual subscription, then what will be the total revenue of the company after the increment?

(b) What will we put to find the maximum profit?

(c) By how much subscription fee the company should increase to have maximum profit?

OR

Find the maximum profit that the company can make if the profit function is given by $P(x) = 41 + 24x - 18x^2$.

37. **Read the text carefully and answer the questions:** **[4]**

In year 2000, Mr. Talwar took a home loan of ₹ 30,00,000 from State Bank of India at 7.5 % p.a. compounded monthly for 20 years.

(a) Find the equated monthly instalment paid by Mr. Talwar.

(b) Find the interest paid by Mr. Talwar in 150th payment.

(c) Find the principal paid by Mr. Talwar in 150th payment.

OR

Find the total interest paid by Mr. Talwar.

[Use $(1.00625)^{240} = 4.4608$, $(1.00625)^{91} = 1.7629$, $(1.00625)^{48} = 1.1187$]

38. An economy produces only coal and steel. The two commodities serve as intermediate inputs in each other's production. 0.4 tonne of steel and 0.7 tonne of coal are needed to produce a tonne of steel. Similarly, 0.1 tonne of steel and 0.6 tonne of coal are required to produce a tonne of coal. If the economy needs 100 tonne of coal and 50 tonne of steel, calculate the gross output of the two commodities. **[4]**

OR

Show that the matrix, $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ satisfies the equation, $A^3 - A^2 - 3A - I_3 = O$. Hence, find A^{-1}.

Solution

Section A

1.

(c) 100

Explanation:

A(adjA) = |A|I

|A|I = 10I

|A| = 10

Now, |adj A| = $|A|^{n-1}$

So, |adj A| = $|A|^{3-1}$

|adj A| = 10^2 = 100

2.

(b) 33

Explanation:

Given n = 34

$\Rightarrow$ degree of freedom (v) = 34 - 1 = 33

3.

(c) ₹ 5935

Explanation:

₹ 5935

4.

(c) Empty

Explanation:

There will be no common region.

5.

(c) 8

Explanation:

8

6.

(c) 12

Explanation:

$p = \frac{1}{4}, \sqrt{npq} = 3$

$\Rightarrow q = \frac{3}{4}$, npq = 9

$\Rightarrow$ Mean = np $= \frac{9}{q}$

$\Rightarrow$ Mean $= 9 \times \frac{4}{3}$ = 12

7. **(a)** 33

Explanation:

n = 100, $p = \frac{1}{3} \Rightarrow q = \frac{2}{3}$

$np = \frac{100}{3} = 33 + \frac{1}{3}$

$\Rightarrow$ Probability is maximum at 33.

8.

(b) one

Explanation:

$\int \frac{1}{y+1} dy = \int \frac{1}{x-1} dx \Rightarrow$ log |y + 1| = log |x - 1| + log C

$\Rightarrow$ y + 1 = C(x - 1)

When x = 0, y = 2

$\therefore$ 3 = C(0 - 1) $\Rightarrow$ C = -3

(y + 1) = -3(x - 1)

Hence, the given differential equation has one solution.

9.

(d) $\frac{2ud}{u^2-v^2}$

Explanation:

upstream speed = (u - v) km/hr

downstream speed = (u - v) km/hr

$t_{\text{downstream}} = \frac{d}{u+v}$

$t_{\text{upstream}} = \frac{d}{u-v}$

$t = t_{\text{downstream}} + t_{\text{upstream}}$

$t = \frac{d}{u+v} + \frac{d}{u-v}$

$t = \frac{(u-v)d+(u+v)d}{(u^2-v^2)}$

$t = \frac{d[u-v+u+v]}{(u^2-v^2)}$

$t = \frac{2ud}{u^2-v^2}$

10.

(b) 20 m

Explanation:

To reach the winning post A will have to cover a distance of (500 - 140) m = 360 m

While A covers 3 m, B covers 4 m.

While A covers 360 m, B covers = $\frac{4\times360}{3}$ = 480 m

$\therefore$ A wins by 20 m.

11. **(a)** 4

Explanation:

a$\equiv$b (mod m)

28 $\equiv$ x (mod 6)

$\Rightarrow \frac{28-x}{6}$ (6 divides (28 - x) completely)

$\Rightarrow$ x = 4

12. **(a)** half-plane that neither contains the origin nor the points on the line 2x + 3y = 6

Explanation:

half-plane that neither contains the origin nor the points on the line 2x + 3y = 6

13.

(b) 21 hours

Explanation:

Let the leak be able to empty the tank in x hours.

Then, amount filled by the pump in one hour = $\frac{1}{3}$

Amount emptied by the leak = $\frac{1}{x}$

Amount actually filled = $\frac{1}{3.5} = \frac{2}{7}$

i.e. $\frac{1}{3} - \frac{1}{x} = \frac{2}{7}$

$\frac{1}{x} = \frac{1}{3} - \frac{2}{7}$

$= \frac{7}{21} - \frac{6}{21} = \frac{1}{21}$
Thus, x = 21 hours

14. **(a)** infinite
Explanation:
Since Z = ax + by has maximum value at two comer points. So, Z has the same maximum value at every point of the line segment joining these two points. Hence, maximum value of Z occurs at infinite points.

15.
(c) $\frac{1}{7}\log\left|\frac{x^7}{x^7+1}\right| + C$
Explanation:
Put $x^7 + 1 = t \Rightarrow 7x^6\,dx = dt \Rightarrow x^6\,dx = \frac{1}{7}\,dt$
$\therefore \int \frac{1}{x(x^7+1)} dx = \int \frac{x^6}{x^7(x^7+1)} dx = \frac{1}{7} \int \frac{1}{(t-1)t} dt$
$= \frac{1}{7} \int \left(\frac{1}{t-1} - \frac{1}{t}\right) dt = \frac{1}{7}(\log(t-1) - \log|t|) + C$
$= \frac{1}{7}\log\left|\frac{x^7}{x^7+1}\right| + C$

16.
(b) Probability of not rejecting H_0 when H_0 is true.

Explanation:
Probability of not rejecting H_0 when H_0 is true.

17. **(a)** $-xe^{-x} + C$
Explanation:
$I = \int (x - 1)e^{-x}$
$= \int xe^{-x}\,dx - \int e^{-x}\,dx$
$= -xe^{-x} - \int 1.(-)e^{-x}\,dx - \int e^{-x}\,dx + c$
$= -xe^{-x} + \int e^{-x}\,dx - \int e^{x}\,dx + c$
$= -xe^{-x} + C$

18. **(a)** Time Series
Explanation:
The organized series of data on the basis of any measure of time is called Time series.

19.
(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
We know that for a non-singular matrix A of order n.
$adj(adj\ A) = |A|^{n-2} A$
and $|adj\ A| = |A|^{n-1}$
$\therefore$ Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

20.
(d) Assertion (A) is false but Reason (R) is true.
Explanation:
Assertion (A) is false but Reason (R) is true.

Section B

21. Construction of 3-yearly moving average

Year	Imported cotton consumption in India	3-yearly moving totals	3-yearly moving averages

	(in '000 bales)		
2010	129	-	-
2011	131	366	122.00
2012	106	328	109.33
2013	91	292	97.33
2014	95	270	90.00
2015	84	272	90.66
2016	93	-	-

22. To calculate the lump sum amount required to provide an annual scholarship of ₹ 3,000, we can use the formula for the present value of a perpetuity. A perpetuity is a series of payments that continues indefinitely.

The formula for the present value of a perpetuity is:

PV = $\frac{Payment}{Interest\ Rate}$

In this case, the annual payment (scholarship) is ₹ 3,000, and the annual interest rate is 5%(0.05 as a decimal). Plug these values into the formula:

PV = $\frac{₹\ 3000}{0.05}$

PV = ₹ 60,000

So, the lump sum amount required to provide an annual scholarship of ₹ 3,000, starting at the end of this year and continuing forever, is ₹ 60,000.

OR

Let P be ₹ x, S.I. = ₹ 1000, r = 10% p.a., n = 4 years

$\therefore \frac{x \times 10 \times 4}{100}$ = 1000

$\Rightarrow$ x = 2500

$\therefore$ Sum = ₹ 2500

23. Let I = $\int_1^2 \frac{\log x}{x^2}$ dx. Then,

I = $\int_1^2 \underset{I}{\log x} \cdot \underset{II}{\frac{1}{x^2}}$ dx = $\left[(\log x)\left(-\frac{1}{x}\right)\right]_1^2 - \int_1^2 \frac{1}{x}\left(-\frac{1}{x}\right)$ dx [Integrating by parts]

I = $\left[-\frac{1}{x}\log x\right]_1^2 - \left[\frac{1}{x}\right]_1^2$ = $(-\frac{1}{2}$ log 2) + (1 $\times$ log 1) - $\left(\frac{1}{2} - \frac{1}{1}\right)$

I = $-\frac{1}{2}$ log 2 + $\frac{1}{2} = \frac{1}{2}$ (-log 2 + 1) = $\frac{1}{2}$ (-log 2 + log e) = $\frac{1}{2}$ log $\left(\frac{e}{2}\right)$

24. (A + 2B)' = A' + (2B)' = A' + 2B' = $\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}'$

= $\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

OR

A = $\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

A^T = $\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$

According to the question matrix A is a skew-symmetric matrix.

$\therefore A^T = -A$

$\Rightarrow \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$

On equating the corresponding elements, we get

a = -2 and b = 3

25. Let the CP. of mixture be ₹ x per kg

Given S.P.= ₹ 47.04 and profit = 12%

∵ S.P. = C.P. + profit

⇒ 47.04 = x + 12% of x

⇒ x = $\frac{4704}{112}$ ⇒ x = 42

Given c = ₹ 35 per kg, d = ₹ 45 per kg, m = ₹ 42 per kg

and quantity of cheaper sugar = 30 kg

So, $\frac{\text{quantity of cheaper sugar}}{\text{quantity of dearer sugar}}=\frac{45-42}{42-35}$

$\Rightarrow \frac{30}{\text{quantity of dearer sugar}}=\frac{3}{7}$

⇒ quantity of dearer sugar = 70 kg

Hence, 70 kg of sugar costing ₹ 45 per kg should be mixed.

Section C

26. Let A_0 be the original amount of radium and A be the amount of radium at any time t. Then, the rate of decomposing of radium is $\frac{dA}{dt}$. It is given that

$\frac{dA}{dt}\propto A$

$\Rightarrow \frac{dA}{dt}=-\lambda A$, where λ is a positive constant

$\Rightarrow \frac{dA}{A}=-\lambda dt$

⇒ log A = -λt + C ...(i)

At t = 0, we have A = A_0. Putting t = 0 and A = A_0 in (i), we get

log A_0 = 0 + C ⇒ C = log A_0

Putting C = log A_0 in (i), we get

log A = -λt + log A_0

$\Rightarrow \log\left(\frac{A}{A_0}\right)=-\lambda t$ ...(ii)

It is given that p% of the original amount of radium disintegrates in l years. This means that the amount of radium present att = l is $\left(A_0-\frac{p}{100}\times A_0\right)=\left(1-\frac{p}{100}\right)A_0$. Putting A = $A_0\left(1-\frac{p}{100}\right)$ and t = l in (ii), we get

$\log\left(1-\frac{p}{100}\right)=-\lambda l\Rightarrow\lambda=-\frac{1}{l}\log\left(1-\frac{p}{100}\right)$

Substituting the value of λ in (ii), we get

$\log\left(\frac{A}{A_0}\right)=\frac{t}{l}\log\left(1-\frac{p}{100}\right)$ ...(iii)

Let A be the amount of radium available after 2l years.

Putting t = 2l in (iii), we get

$\log\left(\frac{A}{A_0}\right)$ = 2 log $\left(1-\frac{p}{100}\right)$

$\Rightarrow \frac{A}{A_0}=\left(1-\frac{p}{100}\right)^2$

$\Rightarrow \frac{A}{A_0}\times 100=\left(1-\frac{p}{100}\right)^2\times 100$ [Multiplying both sides by 100]

$\Rightarrow \frac{A}{A_0}\times 100=\left(10-\frac{p}{10}\right)^2$

OR

The given differential equation can be written as

$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx}$ + Py = Q, where

P = $\frac{2x}{1+x^2}$ and Q = $\frac{4x^2}{1+x^2}$

∴ I.F. = $e^{\int Pdx}=e^{\int\frac{2x}{(1+x^2)}dx}=e^{\log(1+x^2)}$ = 1 + x^2

Multiplying both sides of (i) by I.F. = (1 + x^2), we get

(1 + x^2)$\frac{dy}{dx}$ + 2xy = $4x^2$

Integrating both sides with respect to x, we get

y(1 + x^2) = $\int$ $4x^2$ dx + C [Using: y(I.F.) = $\int$ Q (I.F.) dx + C]

⇒ y (1 + x^2) = $\frac{4x^3}{3}$ + C ...(ii)

It is given that y = 0, when x = 0. Putting x = 0 and y = 0 in (i), we get

0 = 0 + C ⇒ C = 0

Substituting C = 0 in (ii), we get y = $\frac{4x^3}{3(1+x^2)}$, which is the required solution.

27. i. Let the original cost of the washing machine be ₹P and the rate of depreciation be r % p.a. Then the value of machine (in ₹) after one year, two years and 3 years are P(1 - i), $P(1 - i)^2$ and $P(1 - i)^3$ respectively, where i = $\frac{r}{100}$.

According to given,

$P(1 - i) - P(1 - i)^2 = 720$ and $P(1 - i)^2 - P(1 - i)^3 = 648$

⇒ P(1 - i)[1 - (1 - i)] = 720 and $P(1 - i)^2[1 - (1 - i)] = 648$ ...(ii)

⇒ P(1 - i) i = 720 ...(i) and $P(1 - i)^2 \cdot i = 648$

Dividing (ii) by (i), we get

$1 - i = \frac{648}{720} \Rightarrow 1 - i = \frac{9}{10}$

$\Rightarrow i = 1 - \frac{9}{10} \Rightarrow i = \frac{1}{10} \Rightarrow \frac{r}{100} = \frac{1}{10}$

⇒ r = 10

Hence, the rate of depreciation = 10 % p.a.

ii. Putting i = $\frac{1}{10}$ in equation (i), we get

$P\left(1 - \frac{1}{10}\right) \times \frac{1}{10} = 720 \Rightarrow P \times \frac{9}{100} = 720 \Rightarrow P = 8000$

Hence, the original cost of the machine = ₹ 8000

iii. The value of machine at the end of third year = $P(1 - i)^3$

$= 8000\left(1 - \frac{1}{10}\right)^3 = 8000(0.9)^3$

= 8000 × 0.729 = 5832

Hence, the value of the machine at the end of the third year = ₹5832

28. i. MC = 30 + 2x.

As MC = $\frac{dC}{dx}$,

C(x) = $\int$ (MC) dx = $\int$ (30 + 2x) dx

= 30x + x^2 + k, where k is constant of integration.

Given fixed cost (in ₹) = 120 i.e. when x = 0, C(x) = 120

⇒ 30 × 0 + 0^2 + k = 120 ⇒ k = 120.

∴ C(x) = 120 + 30x + x^2

∴ Total cost of producing 100 units = 120 + 30 × 100 + 100^2 = 13120 (in ₹).

ii. Cost of increasing output from 100 to 200 = C(200) - C(100)

= (120 + 30 × 200 + 200^2) - 13120 = 33000 (in ₹).

Alternatively, we can obtain it as

$\int_{100}^{200} (MC)dx = \int_{100}^{200} (30 + 2x)dx = \left[30x + x^2\right]_{100}^{200}$

= (30 × 200 + 200^2) - (30 × 100 + 100^2) = 33000 (in ₹).

29. i. We have, $\sum XP(X) = \frac{1}{2} + \frac{2}{5} + \frac{12}{25} + \frac{2A}{10} + \frac{3A}{25} + \frac{5A}{25} = \frac{25+20+24+10A+6A+10A}{50} = \frac{69+26A}{50}$

Since, $E(X) = \sum XP(X)$

$\Rightarrow 2.94 = \frac{69+26.A}{50}$

$\Rightarrow 26A = 50 \times 2.94 - 69$

$\Rightarrow A = \frac{147-69}{26} = \frac{78}{26} = 3$

ii. We know that,

Var(X) = $E(X)^2 - [E(X)]^2$

$= \sum X^2P(X) - \left[\sum XP(X)\right]^2$

$= \frac{1}{2} + \frac{4}{5} + \frac{48}{25} + \frac{4A^2}{10} + \frac{9A^2}{25} + \frac{25A^2}{25} - [E(X)]^2$

$= \frac{25+40+96+20A^2+18A^2+50A^2}{50} - [E(X)]^2$

$= \frac{161+88A^2}{50} - [E(X)]^2 = \frac{161+88\times(3)^2}{50} - [E(X)]^2 \; [\because A = 3]$

$= \frac{953}{50} - [2.94]^2$ [$\because E(X) = 2.94$]
= 19.0600 - 8.6436 = 10.4164

OR

Let X denote the number of selected scientists who never commit errors in the work and reporting. Clearly, X can take values 0, 1, 2.

P(X = 0) = Probability that two scientists selected commit error either in the work or in reporting

$= \frac{^{10}C_2}{^{30}C_2} = \frac{3}{29}$

P(X = 1) = Probability that one out of two scientists selected does not commit an error in the work and reporting while the other is not so

$= \frac{^{20}C_1 \times ^{10}C_1}{^{30}C_2} = \frac{40}{87}$

P(X = 2) = Probability that two scientists selected do not commit an error in the work and reporting

$= \frac{^{20}C_2}{^{30}C_2} = \frac{38}{87}$

The probability distribution of X is as given below:

X	0	1	2
P(X)	$\frac{3}{29}$	$\frac{40}{87}$	$\frac{38}{87}$

Let $\bar{X}$ be the mean of the distribution. Then,

$\bar{X} = 0 \times \frac{3}{29} + 1 \times \frac{40}{87} + 2 \times \frac{38}{87} = \frac{116}{87}$ = 1.33

This means that on average out of two selected scientists one scientist will not commit errors in the work and reporting.

30.

Years (t_i)	**Sales (y_i)**	**$x_i = t_i - 2015$**	x_i^2	**x_iy_i**
2012	9	-3	9	-27
2013	11	-2	4	-22
2014	13	-1	1	-13
2015	12	0	0	0
2016	14	1	1	14
2017	15	2	4	30
2018	17	3	9	51
n = 7	$\sum y_i$ = 91	$\sum x_i$ = 0	$\sum x_i^2$ = 28	$\sum x_iy_i$ = 33

$a = \frac{\sum y_i}{n} = \frac{91}{7}$ =13

$b = \frac{\sum x_iy_i}{\sum x_i^2} = \frac{33}{28}$ = 1.179

The equation of the straight line trend is

y = ax + b

$\therefore$ y = 13x + 1.179

31. A group of 5 patients treated with medicine A weigh 10, 8, 12, 6, 4 kg. A second group of 7 patients treated with medicine B weigh 14, 12, 8, 10, 6, 2, 11 kg. Comment on the rejection of hypothesis with 5% level of significance.

[Given: $t_{(10,0.05)}$ = 1.812]

Consider,

$H_0 : \mu_1 = \mu_2$ and

$H_1 : \mu_1 > \mu_2$

Where μ_1 and μ_2 denotes population means for the given two groups.

for Medicine A

$\bar{x} = \frac{\sum x}{n} = \frac{40}{5} = 8$

x	10	8	12	6	4	$\sum x$ = 40
$x - \bar{x}$	2	0	4	-2	-4	0
$(x - \bar{x})^2$	4	0	16	4	16	$\sum(x - \bar{x})^2$ = 40

For Medicin B

$\bar{y} = \frac{\sum y}{n} = \frac{63}{7} = 9$

y	14	12	8	10	6	2	11	$\sum y$ = 63
$y - \bar{y}$	5	3	-1	1	-3	-7	2	0
$(y - \bar{y})^2$	25	9	1	1	9	49	4	$\sum (y - \bar{y})^2$ = 98

Now, $S^2 = \frac{1}{n_1+n_2-2}[\sum(x-\bar{x})^2 + \sum(y-\bar{y})^2]$

$S^2 = \frac{1}{5+7-2}[40 + 98]$

$S^2 = \frac{1}{10} \times 138 = 13.8$

S = $\sqrt{13.8}$ = 3.71

$t = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

$t = \frac{8-9}{3.71\sqrt{\frac{1}{5}+\frac{1}{7}}}$

$t = \frac{-1}{3.71\sqrt{\frac{7+5}{35}}}$

$t = \frac{-1}{3.71\sqrt{\frac{12}{35}}}$

$t = \frac{-1}{3.71 \times 0.58}$

t = -0.46

Given: $t_{(10,0.05)}$ = 1.812

Since, $t_{cal.}$value < t_{tab} value

Hence null hypothesis H_0 may be accepted with 5% significance.

Section D

32. Let x number of tennis rackets and y number of cricket bats were sold.

Number of tennis rackets and cricket balls cannot be negative.

Therefore, x $\geq$ 0, y $\geq$ 0

It is given that a tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time.

Also, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftman's time.

Therefore,

1.5x + 3y $\leq$ 42

3 x + y $\leq$ 24

If the profit on a racket and on a bat is ₹20 and ₹10 respectively. Therefore, profit made on x tennis rackets and y cricket bats is ₹20 x and ₹10 y respectively.

Total profit = Z = 20x + 10y

The mathematical form of the given LPP is:

Maximize Z = 20x + 10y

Subject to constraints:

1.5x + 3y $\leq$ 42

3x + y $\leq$ 24

x $\geq$ 0, y $\geq$ 0

First we will convert inequations into equations as follows:

1.5x + 3y = 42, 3x + y = 24, x = 0 and y = 0

Region represented by 1.5x + 3y $\leq$ 42:

The line 1.5x + 3y = 42 meets the coordinate axes at A_1 (28, 0) and B_1 (0, 14) respectively. By joining these points we obtain the line 1.5x + 3y = 42. Clearly (0, 0) satisfies the 1.5x + 3y = 42. So, the region which contains the origin represents the solution set of the inequation 1.5x + 3y $\leq$ 42

Region represented by 3x + y $\leq$ 24:

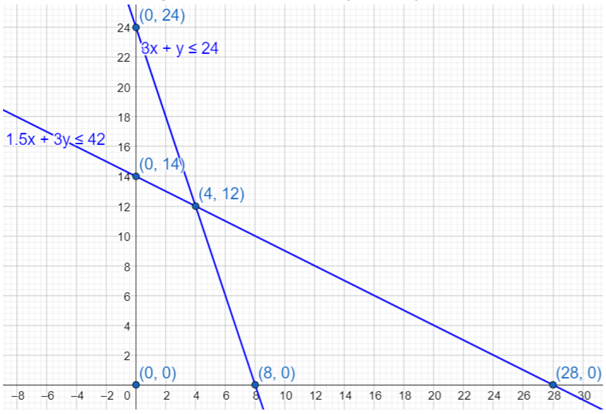
The line 3x + y = 24 meets the coordinate axes at C_1 (8, 0) and D_1 (0, 24) respectively.

By joining these points we obtain the line 3x + y = 24. Clearly (0, 0) satisfies the inequation 3x + y $\leq$ 24. So, the region which contains the origin represents the solution set of the inequation $3x + y \leq 24$.

Region represented by $x \geq 0$ and $y \geq 0$:

Since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$, and $y \geq 0$.

The feasible region determined by the system of constraints $1.5x + 3y \leq 42$, $3x + y \leq 24$, $x \geq 0$ and $y \geq 0$ are as follows.



In the above graph, the shaded region is the feasible region.

The corner points are O(0, 0), B_1(0, 14), E_1(4, 12), C_1(8, 0).

The values of the objective function Z at corner points of the feasible region are given in the following table:

Corner Points	$Z = 20x + 10y$
O(0, 0)	0
B_1(0, 14)	140
E_1(4, 12)	200 $\rightarrow$ Maximum
C_1(8, 0)	160

Clearly, Z is maximum at $x = 4$ and $y = 12$ and the maximum value of Z at this point is 200.

Thus, maximum profit is of ₹200 obtained when 4 tennis rackets and 12 cricket bats were sold.

OR

The above information can be expressed with the help of the following table:

	P	**Q**	**Requirement**
Phosphoric acid	1	2	At least 240
Potash	3	1.5	At least 270
Chlorine	1.5	2	at most 310
Nitrogen	3	3.5	

Let the number of bags of P and Q chosen to be 'x' and 'y' units.

Nitrogen from P = 3x

Nitrogen from Q = 3.5y

Nitrogen form the mixture = 3x + 3.5y

Now,

$\Rightarrow x + 2y \geq 240$

i.e. the minimum requirement of phosphoric acid in the mixture of P and Q is 240 kgs, each of which contains 1 kg and 2 kgs of phosphoric acid respectively

$\Rightarrow 3x + 1.5y \geq 270$

i.e. the minimum requirement of Potash in the mixture of P and Q is 270 kgs, each of which contains 3kgs and 1.5kgs of Potash respectively.

$\Rightarrow 1.5x + 2y \leq 310$

i.e. the maximum requirement of Chlorine in the mixture of P and Q is 310 kgs, each of which contains 1.5 kgs and 2 kgs of Chlorine respectively.

Hence, the mathematical formulation of the LPP is as follows:

Find 'x' and 'y' that

Minimises Z = 3x + 3.5y

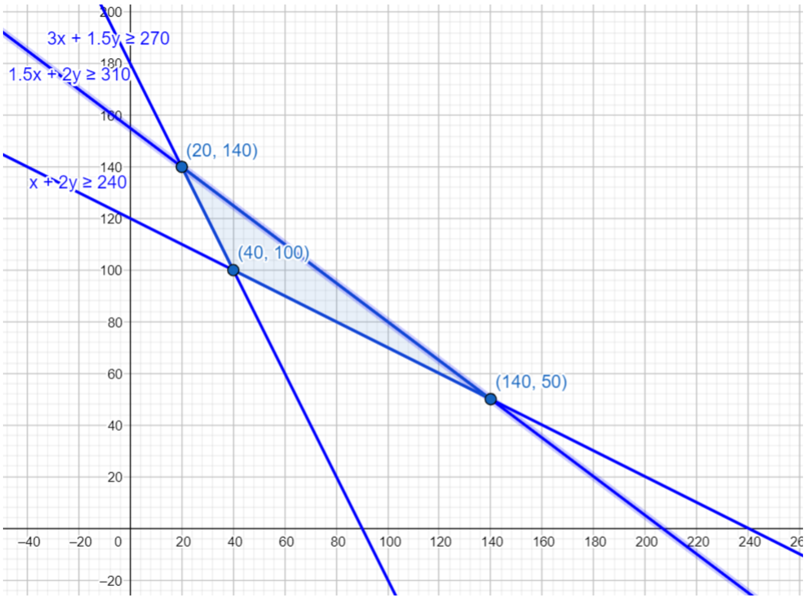
Subject to the following constraints:

i. x + 2y $\geq$ 240

ii. 3x + 1.5y $\geq$ 270

iii. 1.5x + 2y $\geq$ 310

iv. x, y $\geq$ 0 ($\because$ quantity cant be negative)



The feasible region is bounded (ABC)

The corner points of the feasible region is as follows:

Point	**Value of Z = 3x + 3.5y**
A(20, 140)	550
B(40, 100)	470
C(140, 50)	595

Z is minimised at B(40, 100)

The minimum amount of Nitrogen in the mixture is 470 kgs.

33. We have been a week's data

Cost of cassette, C = 300 + $\frac{3}{2}x$

Revenue, R = 2x

Where x = number of cassettes produced and sold in a week.

We know that profit is given by, Profit = Revenue - Cost …(i)

Revenue is the income that a business has from its normal business activities, usually from the sale of goods and services to customers.

A cost is the value of money that has been used up to produce something or deliver a service and hence is not available for use anymore.

And Profit is the gain in the business.

So, it is justified that profit in any business would be measured by the difference in the capital generated by the business and the capital used up in the business.

Profit generated by the company manufacturing cassettes is given by,

Profit = R - C (from (i))

Where, R = Revenue

C = Cost of cassette

Here,

If R < C, then

Profit < 0

$\Rightarrow$ There is a loss.

If R = C, then

Profit = 0

$\Rightarrow$ There is no profit no loss.
If R > C, then
Profit > 0
$\Rightarrow$ There is a profit.
We need to find the number of cassettes sold to make a profit. That is, we need to find x.
So, R > C (to realize a profit)
Substituting values of R and C. We get
2x > 300 + $\frac{3}{2}x$
$\Rightarrow$ 2x - $\frac{3}{2}x$ > 300
$\Rightarrow \frac{4x-3x}{2} > 300$
$\Rightarrow \frac{x}{2} > 300$
$\Rightarrow$ x > 300 $\times$ 2
$\Rightarrow$ x > 600
This means that x must be greater than 600.
Thus, the company must sell more than 600 cassettes to realize a profit.

34. Given, μ = 80 km/h , σ = 10 km/h
  1. P(X < 60) = P $\left(Z < \frac{60-80}{10}\right)$ = P(Z < -2)
     = F(-2) = 1 - F(2) = 1 - 0.9772 = 0.0228
  2. P(X > 100) = P$\left(Z > \frac{100-80}{10}\right)$ = P(Z > 2)
     = 1 - P(Z $\leq$ 2) = 1 - F(2)
     = 1 - 0.9772 = 0.0228
  3. P(90 < X < 110) = P $\left(\frac{90-80}{10} < Z < \frac{110-80}{10}\right)$
     = P(1 < Z < 3) = F(3) - F(1)
     = 0.9986 - 0.8413 = 0.1573

OR

Total number of balls = 5 + 2 = 7
Two balls are drawn at random.
Let X denote the number of black balls drawn, then X can take values 0, 1, 2.
P(X = 0) = P(no black ball) = $\frac{^5C_2}{^7C_2} = \frac{5.4}{7.6} = \frac{10}{21}$
P(X = 1) = P(one black ball) = $\frac{^5C_1 \times {}^2C_1}{^7C_2} = \frac{10}{21}$
P(X = 2) = P(two black balls) $= \frac{^2C_2}{^7C_2} = \frac{1}{21}$.
$\therefore$ Probability distribution of the number of black balls drawn is $\begin{pmatrix} 0 & 1 & 2 \\ \frac{10}{21} & \frac{10}{21} & \frac{1}{21} \end{pmatrix}$.
We construct the following table:

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{10}{21}$	0	0
1	$\frac{10}{21}$	$\frac{10}{21}$	$\frac{10}{21}$
2	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{4}{21}$
Total		$\frac{12}{21}$	$\frac{14}{21}$

Mean $= \Sigma p_i x_i = \frac{12}{21} = \frac{4}{7}$.
Variance $\Sigma p_i \dot{x}_i^2 - (\Sigma p_i x_i)^2 = \frac{14}{21} - \left(\frac{4}{7}\right)^2$
$= \frac{2}{3} - \frac{16}{49} = \frac{98-48}{147} = \frac{50}{147}$.

35. To determine whether Mrs. Jain should buy the bond, we need to calculate the bond's yield to maturity (YTM) and compare it to her minimum required rate of return of 15%. The YTM represents the total return an investor can expect to receive if they hold the bond until maturity.
First, let's calculate the YTM using the given information:

Current Bond Price (P) = ₹ 870

Par Value (F) = ₹ 1,000

Coupon Payment (C) = ₹ 1,000 $\times$ 11% = ₹ 110 per year

Number of Years to Maturity (n) = 5

Using the formula for YTM, we need to find the discount rate (YTM) that makes the present value of the bond's cash flows equal to the current price:

P = $\frac{C}{(1+YTM)^1} + \frac{C}{(1+YTM)^2} + \ldots + \frac{C+F}{(1+YTM)^n}$

870 = $\frac{110}{(1+YTM)^1} + \frac{110}{(1+YTM)^2} + \frac{110}{(1+YTM)^3} + \frac{110}{(1+YTM)^4} + \frac{110+1000}{(1+YTM)^5}$

You've already been given that $(1.15)^{-5}$ = 0.4971. You can use this value to simplify the equation:

870 = $\frac{110}{1+YTM} + \frac{110}{(1+YTM)^2} + \frac{110}{(1+YTM)^3} + \frac{110}{(1+YTM)^4} + \frac{110+1000}{0.4971}$

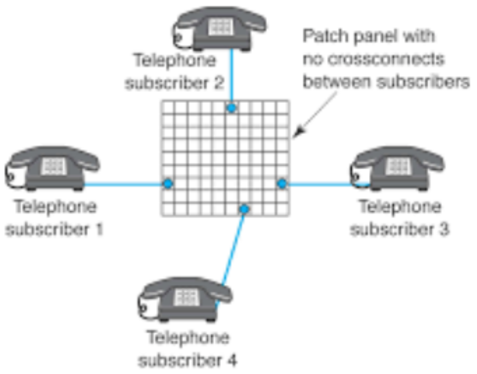
Now, solve for YTM using this equation. You can use numerical methods or financial calculators to find YTM. When you solve for YTM, you'll find that it's approximately 19.72%.

Since the calculated YTM (19.72%) is greater than Mrs. Jain's minimum required rate of return (15%), the bond appears to offer a yield higher than her required return. Therefore, she should consider buying the bond, as it seems to provide a potentially favorable return relative to her investment criteria. However, she should also consider other factors such as her risk tolerance and investment goals before making a decision.

Section E

36. **Read the text carefully and answer the questions:**

A telephone company in a town has 500 subscribers on its list and collects fixed charges of 300 per subscriber per year. The company proposes to increases the annual subscription and it is believed that for every increase of ₹ 1 one subscriber will discontinue the service.



(i) R(x) = $-x^2 + 200x + 150000$

(ii) R'(x) = 0

(iii) ₹ 100

OR

49

37. **Read the text carefully and answer the questions:**

In year 2000, Mr. Talwar took a home loan of ₹ 30,00,000 from State Bank of India at 7.5 % p.a. compounded monthly for 20 years.

(i) Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ =0.00625,

n = 12 $\times$ 20 = 240 months.

EMI = $\frac{3000000 \times 0.00625(1.00625)^{240}}{(1.00625)^{240}-1}$

$= \frac{3000000 \times 0.00625 \times 4.4608}{3.4608}$ = ₹ 24167.82.

(ii) Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ =0.00625,

n = 12 $\times$ 20 = 240 months.

Interest paid in 150th payment = $\frac{\text{EMI}\left[(1+i)^{240-150+1}-1\right]}{(1+i)^{240-150+1}} = \frac{24167.82\left[(1.00625)^{91}-1\right]}{(1.00625)^{91}}$

$= \frac{24167.82 \times 0.7629}{1.7629}$ = ₹ 10458.69

(iii) Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ =0.00625,

n = 12 $\times$ 20 = 240 months.

Principal paid in 150th payment = EMI - Interest paid in 150th payment

=₹ 24167.82 - ₹ 10458.69 = ₹ 13709.13.

OR

Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ =0.00625,

n = 12 $\times$ 20 = 240 months.

Total interest paid = n $\times$ EM! - P = 240 $\times$ 24167.82 - 30,00,000 = ₹2800276.80

38. The input-output coefficients may be arranged in the following tabular form:

INPUT-OUTPUT TABLE

Output		Input	
		Steel	Coal
	Steel	0.4	0.1
	Coal	0.7	0.6

Let A denote the input-output coefficient matrix or the technology matrix. Then

A = $\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

The Leontief matrix is given by

I - A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{bmatrix}$

$\therefore$ |I - A| = $\begin{vmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{vmatrix}$ = 0.24 - 0.07 = 0.17

We find that |I - A| > 0 and diagonal elements of A are all less than 1. So, Hawkins-Simon conditions are satisfied and hence the system is viable.

Let the gross output of steel and coal be x_1 units and x_2 units respectively to meet the final demand given by the demand vector D = $\begin{bmatrix} 50 \\ 100 \end{bmatrix}$. Further, let X = $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Then,

X = $(I - A)^{-1}$D ...(i)

Now,

I - A = $\begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{bmatrix}$

$\Rightarrow$ adj (I - A) = $\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

$\therefore$ $(I - A)^{-1}$ = $\frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

Substituting the values of $(I - A)^{-1}$ and D in (i), we obtain

X = $\frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}\begin{bmatrix} 50 \\ 100 \end{bmatrix} = \frac{1}{0.17}\begin{bmatrix} 20+10 \\ 35+60 \end{bmatrix} = \begin{bmatrix} \frac{30}{0.17} \\ \frac{95}{0.17} \end{bmatrix} = \begin{bmatrix} 176.5 \\ 558.8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 176.5 \\ 558.8 \end{bmatrix}$ $\Rightarrow$ x_1 = 176.5, x_2 = 558.8

Hence, the gross outputs of steel and coal, for the given demand, are 176.5 and 558.8 tonnes respectively.

OR

Here, we have:

A = $\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$A^3 = A^2A$

A^2 = $\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

= $\begin{bmatrix} 1+0-6 & 0+0-8 & -2+0-2 \\ -2+2+6 & 0+1+8 & 4-2+2 \\ 3-8+3 & 0-4+4 & -6+8+1 \end{bmatrix} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$

$A^2A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5^2+16-12 & 0-8+16 & 10-16-4 \\ 6-18+12 & 0-9+16 & -12+18+4 \\ -2-0+9 & 0-0-12 & 4+0+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix}$

Now, $A^3 - A^2 - 3A - I$

$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1+5 & -8+8 & -10+4 \\ 0-6 & 7-9 & 10-4 \\ 7+2 & 12-0 & 7-3 \end{bmatrix} + \begin{bmatrix} -3-1 & -0-0 & 6-0 \\ 6-0 & +3-1 & -6-0 \\ -9-0 & -12+0 & -3-1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 & -6 \\ -6 & -2 & 6 \\ 9 & 12 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 6 \\ 6 & 2 & -6 \\ -9 & -12 & -4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Thus, $A^3 - A^2 - 3A - I = 0$

Multiply both sides by A^{-1}, we get

$A^{-1}A^3 - A^1A^2 - 3A^{-1}A - IA^{-1} = 0$

$A^2 - A - 3I = A^{-1}$ ...(since $A^{-1}A = I$)

$\Rightarrow A^{-1} = (A^2 - A - 3I)$

$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} -5-1-3 & -8-0-0 & -4+2-0 \\ 6+2-0 & 7+1-3 & 4-2-0 \\ -2-3-0 & 0-4-0 & 3-1-3 \end{bmatrix}$

$= \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$